

**QUESTION 2016**

Group - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following:

i) The order and degree of the differential equation  $\sqrt{\frac{dy}{dx} + \frac{d^2y}{dx^2}} + x = 1$  is

✓ a) (2, 2)

b) (1, 2)

c)  $\left(1, \frac{1}{2}\right)$

d)  $\left(\frac{1}{2}, 1\right)$

ii) Family of curves  $y = e^x (A \cos x + B \sin x)$  is represented by the differential equation

a)  $\frac{d^2y}{dx^2} = \frac{2dy}{dx} - y$

✓ b)  $\frac{d^2y}{dx^2} = \frac{2dy}{dx} - 2y$

c)  $\frac{d^2y}{dx^2} = \frac{dy}{dx} - 2y$

d)  $\frac{d^2y}{dx^2} = \frac{2dy}{dx} - y$

iii) Integrating factor of  $x \frac{dy}{dx} - y = 1$  is

a) x

✓ b)  $\frac{1}{x}$

c) -x

d)  $-\frac{1}{x}$

iv) The differential equation which has the singular solution is

a)  $\frac{dx}{dy} = \frac{x}{y-3}$

b)  $\frac{dy}{dx} - 3y = y$

c)  $\frac{dy}{dx} - y = x$

d)  $\frac{dy}{dx} - 2xy = x^2$

Answer: none of these

v) The P.I. of  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^x$  is

a)  $\frac{e^x}{3}$

b)  $\frac{e^x}{2}$

c)  $\frac{e^x}{6}$

✓ d) none of these

vi) The auxiliary equation of the differential equation  $\frac{d^2y}{dx^2} = \sin x$  is

a)  $y = \cos 2x + \sin 2x$

b)  $y = c_1 \cos 2x + c_2 \sin 2x$

c)  $y = c_1 \cos x + c_2 \sin 2x$

✓ d) none of these

vii) The sequence  $\{(-1)^n\}$  is

- a) convergent      ✓ b) oscillatory      c) divergent      d) none of these

viii) If the vectors  $(5, 2, 3), (7, 3, a), (9, 4, 5)$  of a vector space  $R^3$  over  $R$  be linearly independent, then the value of  $a$  is not equal to

- ✓ a) 2      b) 3      c) 1      d) 0

ix) If  $T(x, y, z) = (x, y, 0)$  for all  $(x, y, z) \in R^3$  is a linear transformation, then kernel of  $T$  is

- a)  $(0, 0, 0)$       b) X-axis      c) Y-axis      ✓ d) Z-axis

x)  $T$  is a transformation from  $R_2$  to  $R_3$  defined by  $T(x_1, x_2) = (x_1, x_1^2 + x_2, -x_1)$ , then the image of  $(1, 2)$  is

- a)  $(1, 1, 1)$       b)  $(0, 3, -1)$       ✓ c)  $(1, 3, -1)$       d)  $(0, 0, 0)$

xi) If  $(3, 1) = x(1, 2) + y(0, 3)$ , then the values of  $x$  and  $y$  are respectively

- a)  $3, -5$       b)  $3, 1$       ✓ c)  $3, -\frac{5}{3}$       d)  $3, -\frac{5}{2}$

xii) Union of two subspaces of a vector is a

- a) subspace of the vector space      ✓ b) not a subspace of the vector space  
c) none of these

### Group - B

#### (Short Answer Type Questions)

2. What is an exact differential equation? If  $\left(y + \frac{1}{x} + \frac{1}{x^2 y}\right) dx + \left(x - \frac{1}{y} + \frac{A}{xy^2}\right) dy = 0$  is exact, then find the value of  $A$ .

See Topic: DIFFERENTIAL EQUATIONS, Short Answer Type Question No. 10.

3. Solve the differential equation  $(D^3 - 2D^2 - 5D + 6)y = e^{4x}$ .

See Topic: DIFFERENTIAL EQUATIONS, Long Answer Type Question No. 11.

4. Define a monotonic increasing sequence and a strictly monotonic decreasing sequence.

Show that the sequence  $\left\{\left(1 + \frac{1}{n}\right)^n\right\}$  is monotonic increasing sequence.

See Topic: SEQUENCE, Short Answer Type Question No. 7.



5. Test the convergence of the series  $\sum_{n=1}^{\infty} \{\sqrt{n^2+1} - n\}$

See Topic: SERIES, Short Answer Type Question No. 15.

6. What is a coordinate vector? What is the coordinate vector of  $(3, 5, -2) \in R^3$  relative to the basis  $\{(1, 2, 3), (2, 0, 1), (1, -1, 0)\}$ ?

See Topic: LINEAR ALGEBRA, Short Answer Type Question No. 23.

### Group - C

(Long Answer Type Questions)

7. a) Explain degree and order of a differential equation with example.

b) Find the general and singular solutions of  $y + xp - p^2 x^4 = 0$ , where  $p = \frac{dy}{dx}$ .

c) Show that the sequence  $\left\{2 + \frac{(-1)^n}{x}\right\}$  is convergent.

a) & b) See Topic: DIFFERENTIAL EQUATIONS, Long Answer Type Question No. 18(a) & (b).

c) See Topic: SEQUENCE, Long Answer Type Question No. 7.

8. a) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x^2 + e^{3x}, \text{ given that } y = 0, \frac{dy}{dx} = 2, \text{ when } x = 0.$$

b) Show that the sequence  $\sqrt{7}, \sqrt{7+\sqrt{7}}, \sqrt{7+\sqrt{7+\sqrt{7}}}$  converges to 7.

c) If  $T: R^3 \rightarrow R^3$  is given by  $T(x, y, z) = (x - 2y, y - 2z, z - 2x)$ , for  $(x, y, z) \in R^3$ , obtain the matrix representation for linear transformation T.

a) See Topic: DIFFERENTIAL EQUATIONS, Long Answer Type Question No. 19.

b) See Topic: SEQUENCE, Long Answer Type Question No. 8.

c) See Topic: LINEAR ALGEBRA, Long Answer Type Question No. 3.

9. a) State Leibnitz theorem for alternating series and test the convergence of the series

$$\frac{1}{1} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

b) Test the convergence of the following series  $\frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{2^3} + \frac{1}{3^3} + \dots$

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c) Define a basis of a finite dimensional vector space  $V$ . Show that  $W = \{(x, y, z) \in R^3 : x + 2y - z = 0, 2x - y + 3z = 0\}$  is a subspace. Find dimension and a basis of  $W$ .

a) & b) See Topic: SERIES, Long Answer Type Question No. 9(a) & (b).

c) See Topic: LINEAR ALGEBRA, Short Answer Type Question No. 2 & Long Answer Type Question No. 23.

10. a) Define linearly dependence of a set of finite number of vectors. Find the value of  $x$  such that the vectors  $(1, 2, 1)$ ,  $(x, 3, 1)$  and  $(2, x, 0)$  are linearly dependent.

b) Show that intersection of two subspaces is a subspace.

c) Find the integrating factor of the differential equation  $x \cos x dy + y(x \sin x + \cos x) = dx$ .

a) See Topic: LINEAR ALGEBRA, Long Answer Type Question No. 1.

b) See Topic: LINEAR ALGEBRA, Short Answer Type Question No. 21.

c) See Topic: DIFFERENTIAL EQUATIONS, Long Answer Type Question No. 20.

11. a) Solve  $(2x+3)^2 \frac{d^2y}{dx^2} - (2x+3) \frac{dy}{dx} - 12y = 6x$ .

b) Obtain the differential equation for  $x^2 + y^2 + 2gx + 2fy + c = 0$ .

c) Test the convergence of the following series:

$$1 + \frac{2^2}{3^2}x + \frac{2^2 \cdot 4^2}{3^2 \cdot 5^2}x^2 + \frac{2^2 \cdot 4^2 \cdot 6^2}{3^2 \cdot 5^2 \cdot 7^2}x^3 + \dots (x \neq 1).$$

a) & b) See Topic: DIFFERENTIAL EQUATIONS, Long Answer Type Question No. 21(a) & (b).

c) See Topic: SERIES, Long Answer Type Question No. 12.